MA114 Summer 2018
Worksheet 9a – Infinite Series – 6/25/18

1. Identify the following statements as true or false and explain your answers.

   (a) If the sequence of partial sums of an infinite series is bounded then the series converges.

   False. $\sum_{n=0}^{\infty} (-1)^n$ is a counterexample.

   (b) $\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} a_n$ if the series converges.

   False. $\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$, but $\lim_{n\to\infty} \frac{1}{2^n} = 0$.

   (c) $\sum_{n=1}^{\infty} a_n = \sum_{n=0}^{\infty} a_{n+1}$ if both series converge.

   True. This is just re-indexing terms: $a_1 + a_2 + a_3 + a_4 + \dots = a_{0+1} + a_{1+1} + a_{2+1} + a_{3+1} + \dots$

   (d) Every infinite series with only finitely many nonzero terms converges.

   True. Eventually, we are just adding 0, which will not change the sum.

   (e) A finite number of terms of an infinite series can be changed without affecting whether or not the series converges.

   True. ex: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = 1$, so changing the first term to $\frac{5}{2}$ gives $\frac{5}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 2 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 3$.

2. Find the first four partial sums of the series

$$\sum_{n=0}^{\infty} \frac{1}{n+3} - \frac{1}{n+5}$$

and find its sum S.

$S_0 = \frac{1}{3} - \frac{1}{5}$ $\quad$ $S_1 = \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{1}{6}\right)$ $\quad$ $S_2 = \left(\frac{1}{3} - \cancel{\frac{1}{5}}\right) + \left(\frac{1}{4} - \frac{1}{6}\right) + \left(\cancel{\frac{1}{5}} - \frac{1}{7}\right)$

$= \frac{1}{3} + \frac{1}{4} - \frac{1}{6} - \frac{1}{7}$

$S_3 = \left(\frac{1}{3} - \cancel{\frac{1}{5}}\right) + \left(\frac{1}{4} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{5}} - \frac{1}{7}\right) + \left(\cancel{\frac{1}{6}} - \frac{1}{8}\right)$

$= \frac{1}{3} + \frac{1}{4} - \frac{1}{7} - \frac{1}{8}$

$\vdots$

$S_N = \frac{1}{3} + \frac{1}{4} - \frac{1}{N+4} - \frac{1}{N+5}$

$S = \lim_{N\to\infty} S_N = \frac{1}{3} + \frac{1}{4} - 0 - 0 = \frac{7}{12}$

3. For each of the following series, determine whether the series converges or diverges. If it converges, find what value it converges to.

(a) $\sum_{n=0}^{\infty} \frac{2+3^n}{5^n}$ $= \sum_{n=0}^{\infty} \frac{2}{5^n} + \sum_{n=0}^{\infty} \frac{3^n}{5^n}$

geom. with $a=2$, $r=\frac{1}{5}$ geom. with $a=1$, $r=\frac{3}{5}$

$= \frac{2}{1-\frac{1}{5}} + \frac{1}{1-\frac{3}{5}}$

$= \frac{5}{2} + \frac{5}{2} = 5$

(b) $\sum_{n=0}^{\infty} \frac{2n+3}{4n+1}$ Diverges by the Divergence Test:

$\lim_{n\to\infty} \frac{2n+3}{4n+1} \overset{L'H}{=} \frac{2}{4} = \frac{1}{2}$

(c) $\sum_{n=0}^{\infty} (-1)^{n-1} \frac{n}{n+1}$ Diverges by the Divergence Test:

$\lim_{n\to\infty} (-1)^{n-1} \frac{n}{n+1}$ DNE b/c it approaches +1 for odd n and -1 for even n.

(d) $\sum_{n=1}^{\infty} e^{-n}$. Reindex to use geometric series formula:

$= \sum_{k=0} e^{-(k+1)} = \sum_{k=0} \frac{1}{e}\left(\frac{1}{e}\right)^k$ $a=\frac{1}{e}$, $r=\frac{1}{e}$

$k=n-1$
$n=k+1$

$= \frac{1/e}{1-1/e} = \boxed{\frac{1}{e-1}}$